RAN-2203000205023006

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T. Y. B. Sc. (Sem. - V) Examination March - 2023

Mathematics : MTH - 506

Number Theory - I

Time: 2 Hours] **[Total Marks: 50**

સૂચના : / Instructions

(1) નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fill up strictly the details of ☛ signs on your answer book

Name of the Examination:

☛ **T. Y. B. Sc. (Sem. - V)**

Name of the Subject :

☛ **Mathematics : MTH - 506 Number Theory - I**

Subject Code No.: 2203000205023006

Seat No.:

Student's Signature

(2) All questions are compulsory.

(3) Follow usual notations.

(4) Figures to the right indicate marks of the question.

Que 1: Answer Any Five from the following: **(10)**

(1) If gcd $(a,b) = 3$ then show that gcd $(a/3, b/3) = 1$.

(2) If a/c and b/c with gcd $(a,b) = 1$ then prove that ab/c.

(3) Solve: $81x + 6y = 29$.

(4) Check whether the integer 509 is a prime or composite?

(5) Prove that the only prime of the form $n^3 - 1$ is 7.

(6) Find the prime factorization of the integer 2022.

(7) For any integer a, prove that $a^4 \equiv 0$ or $1 \pmod{5}$.

(8) Find the remainder when 41^{65} is divided by 7.

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Que 2: **Answer any two questions:** **(10)**

(1) For positive integers a and b prove that gcd (a,b) 1cm $(a,b) = a.b$.

(2) Find integers x and y such that gcd (741, 1079) = $741x + 1079y$.

(3) If a is odd integer, then prove that $32 \mid (a^2 + 3)(a^2 + 7)$.

Que 3: **Answer any two questions:** **(10)**

(1) If a is odd integer then show that $(3a, 3a + 2) = 1$.

(2) Determine all the solutions in the integers of the Diophantine eqn. $123x + 360y = 99$.

(3) Show that the linear Diophantine equation $ax + by = c$ has a solution if and only if $d \mid c$ where $d = \gcd(a, b)$.

Que 4: **Answer any two questions:** **(10)**

(1) Prove that any composite number a will always have a prime divisor p such that $p \leq \sqrt{a}$.

(2) If $p \geq q \geq 5$ and p & q both are primes then show that $24 \mid p^2 - q^2$.

(3) Prove that any prime of the form $3n + 1$ is also of the form $6m + 1$.

Que 5: **Answer any two questions:** **(10)**

(1) If $a \equiv b \pmod{n}$ and $c \equiv d \pmod{n}$ then prove that $a + c \equiv b + d \pmod{n}$ and $ac \equiv bd \pmod{n}$.

(2) Find the reminder when 5555^{5555} is divided by 9.

(3) Working modulo 9 or 11, find the missing digit x in the following calculation :
$1548 \times 3520 = 5448x60$.
